International Journal of Education and Science Research Review
Volume-1, Issue-6 December- 2014
E- ISSN 2348-6457
P- ISSN2349-1817
www.ijesrr.org Email- editor@ijesrr.org

Effect of Basic Set Operations on Graph Theory

Mohd Khursheed Mir
Mewar University
Rajasthan

ABSTRACT:
Graph theory has strong historical roots in Mathematics. Its birth is usually associated with the four-color problem posed by Francis Guthrie in 1852, but its real origin probably goes back to the Seven Bridges of Konigsberg Problem proved by Leonhard Euler in 1736. In general, a graph is a nonempty set of points (vertices) and the most basic information preserved by any graph structure refers to adjacency relationships (edges) between some pairs of points. In the simplest graphs, edges do not have to hold any attributes, except their endpoints, but in more sophisticated graph structures, edges can be associated with a direction or assigned a label. There are several reasons for the acceleration of interest in graph theory. Graph theory has aspects that connect it with other areas of Mathematics such as algebra, geometry, topology, numerical analysis, matrix theory, and combinatory and so on. The expressive power of the graph models placing special emphasis on connectivity between objects has made them the models of choice in chemistry, physics, biology, and other fields. Graph theory is a young but rapidly maturing subject. Within the quarter of a century, concurrent with the growth of such areas as computer science, electrical and computer engineering, and operations research, graph theory has seen explosive growth. Perhaps the fastest-growing area within graph theory is the study of domination, topological graph theory, coloring theory, and related subset problems.

Key words: Graph theory, algebra, geometry, topology

INTRODUCTION

We know that operations have an important role in Mathematics. From arithmetic we know that +, –, ×, ÷ are operations, which can be defined on elements as follows:

(i) a + b is defined as sum of two numbers a and b.

(ii) a – b is defined as difference of two numbers a and b.

(iii) ab is defined as product of two numbers a and b.

(iv) a/b is defined as division of two numbers a and b.

In a similar fashion we will define the operations of union, intersection, difference, product complement in case of sets.

Here in this chapter we are going to define all the above mentioned operations on the edge set defined on the simple graph.

OPERATIONS ON GRAPHS

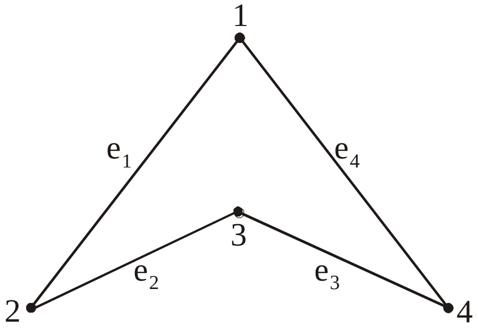
Union of Edge Set:

The union of two edge set E_1 & E_2 denoted by $E_1 \cap E_2$ is the set of all edges which are member of E_1 or E_2 (or both) where E_1 and E_2 are two subsets of the edge set E of the simple graph G.

i.e. $E_1 \cup | E_2 = \{e : e \in E_1 \text{ or } e \in E_2\}$

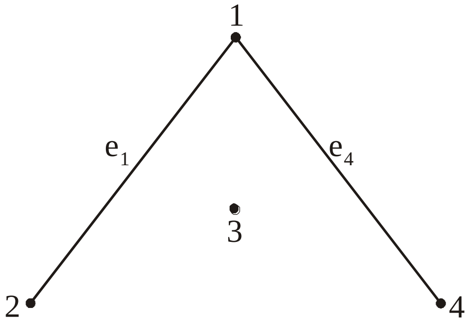
Example: let us assume that there exists a simple graph.

G (V, E) where E = {e_1, e_2, e_3, e_4}.

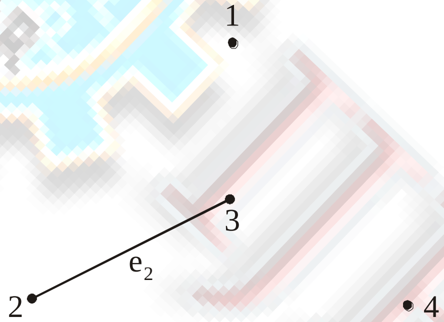


Let there exist $E_1, E_2 \subset E$ s.t.

E_1 = {e_1, e_4} i.e.

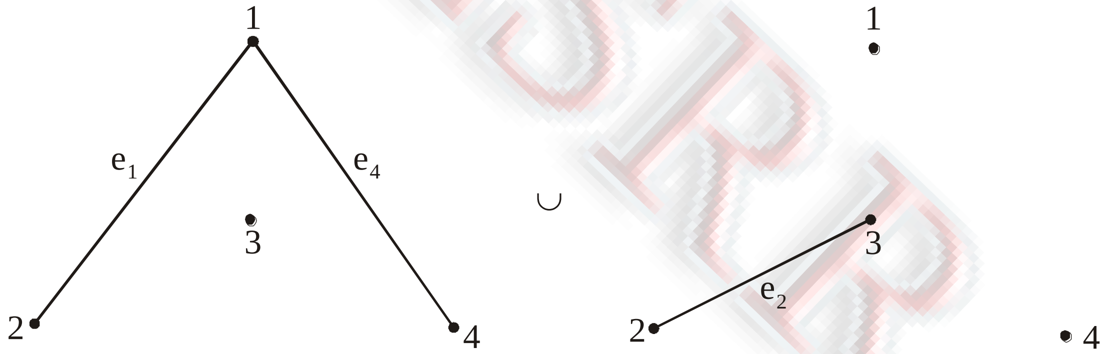


and E_2 = {e_2} i.e.

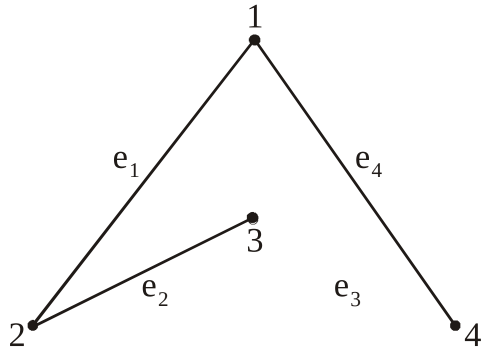


Then the union of these two edges set is:

$E_1 \cup E_2$ =



=



Similarly, we can more clarify the above concept with the help of this example.

Let us consider an example, there exist a simple graph G(V, E), where E = {e_1, e_2, e_3, e_4, e_5}

Intersection of Edge Set:

The intersection of two edge set E_1 and E_2 is the set of all the edges which belongs to E_1 and E_2 and is denoted by $E_1 \cap E_2$.

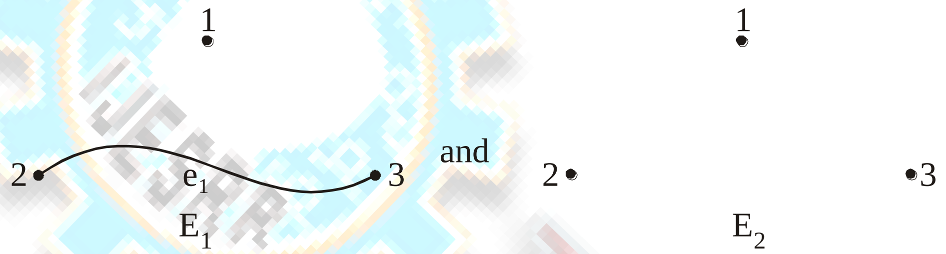
Thus $E_1 \cap E_2 = \{e : e \in E_1 \text{ and } e \in E_2\}$

Example: let there is a graph G defined on the vertex set V and edge set E.

Where E = {e_1}

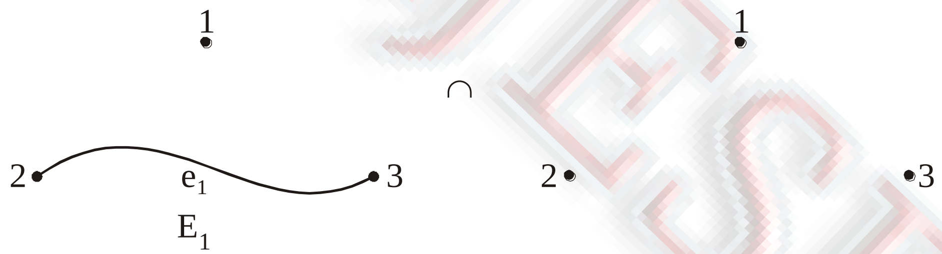
Let E_1 = {e_1} and E_2 = { }

i.e.

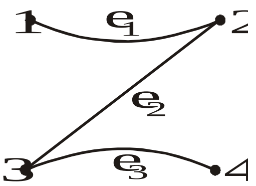


hence the intersection of these two sets are :

$E_1 \cap E_2$ =



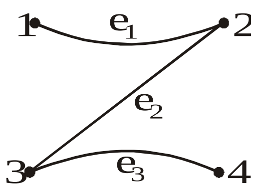
=



Hence, the intersection of null set with any edge set is always a null set likewise the set of elements.

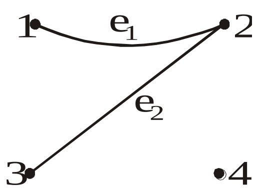
Example: Given a graph G(V,E) such that V is the set of vertex and E is the edge set on G.

let E = {e_1, e_2, e_3} i.e.



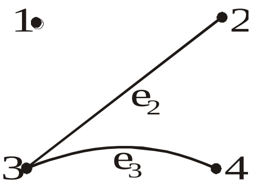
Here E_1 and E_2 are two subset of E.

$E_1 = \{e_1, e_2\}$ i.e.



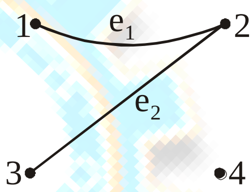
and

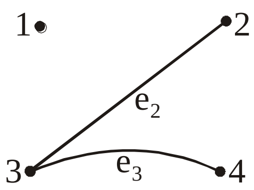
$E_2 = \{e_2, e_3\}$ i.e.

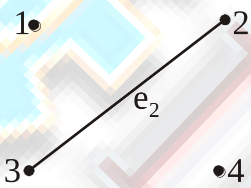


The intersection of these two sets E_1 & E_2 is :

$E_1 \cap E_2 =$







Hence the intersection of the sets E_1 & E_2 is $\{e_2\}$.

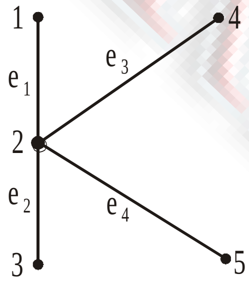
Difference of Edge Set:

If the two edge set E_1 and E_2 of the graph G, then the set consisting of the edges which belongs to E_1, but not the edges of E_2, is said to be the difference of the edge set E_1 and E_2, denoted by $E_1 - E_2$.

i.e. $E_1 - E_2 = \{e : e \in E_1, \text{ but } e \notin E_2\}$.

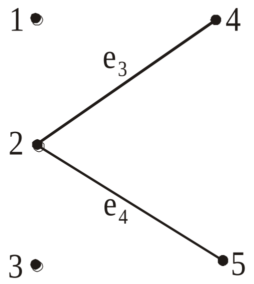
Example: E is an edge set on the given graph G, such that $E = \{e_1, e_2, e_3, e_4\}$.

let E_1 and E_2 two edge subset of the edge.

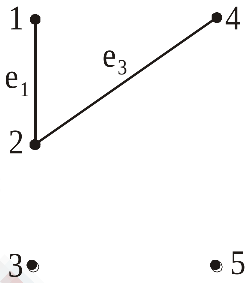


E

$E_1 = \{e_3, e_4\}$ i.e.



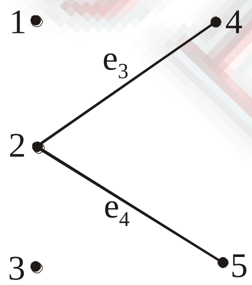
and



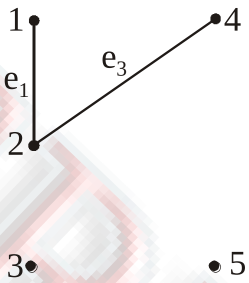
E_2 is defined as $E_2 = \{e_1, e_3\}$

Now we find the difference of the edge set E_1 and E_2.

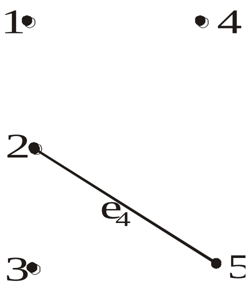
$E_1 - E_2 =$



–



=

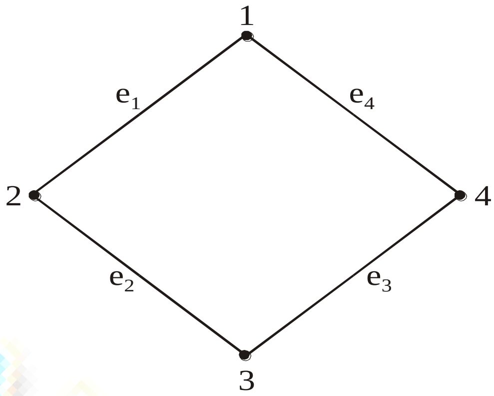


It shows that E_1 has only edge e_4 which is not present in E_2. Thus $E_1 - E_2$ contains only one edge e_4 as similar of the set of elements.

Complement : The complement of the edge set E is denoted by $\bar{E}$, is the graph on the same number of vertices of E obtained by deleting all the edges which are in E and by adding all the edges which are not in E by $\bar{E} = \cup - E$. where $\cup$ is the maximum edge set.

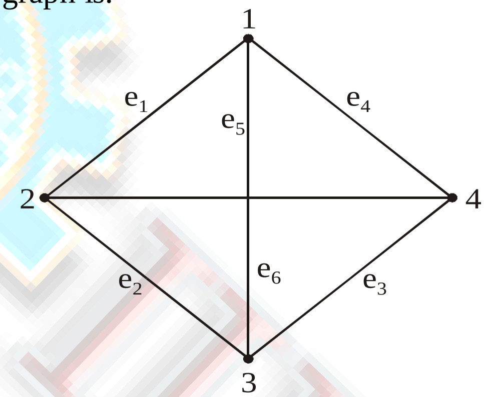
Example: let G (V, E) be a simple graph and E be the edge set of the graph G. Then find the complement of the edge set E i.e. $\bar{E}$.

let E = {e_1, e_2, e_3, e_4}

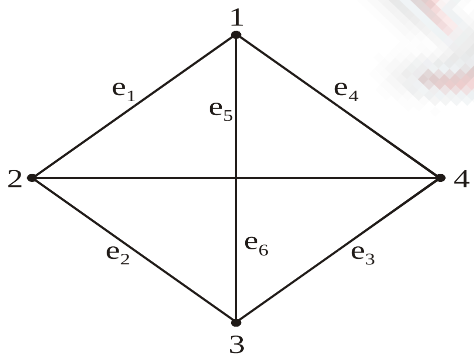


E

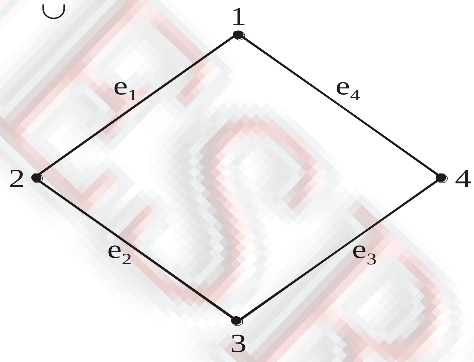
Now to find the complement of the above given edge set E. We have to find the complement of E and for the sake find the difference of E with the compete graph on the same set of vertices.
Thus the complete graph or the full graph is:
Thus on taking the difference:



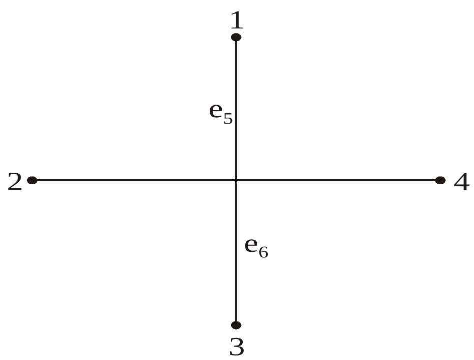
∪



∪



E



E

Thus it reflects that the complement of an edge set works in the similar manner as of the set of elements.

Ring sum of the Edge sets:

Let E_1 and E_2 are two edges set then the ring sum of edges set denoted by $E_1 \oplus E_2$ is containing all the edges which are not common in both.

$E_1 \oplus E_2 = \{e : e \in | E_1 \text{ or } \in E_2 \text{ but } e \notin E_1 \cap E_2\}$

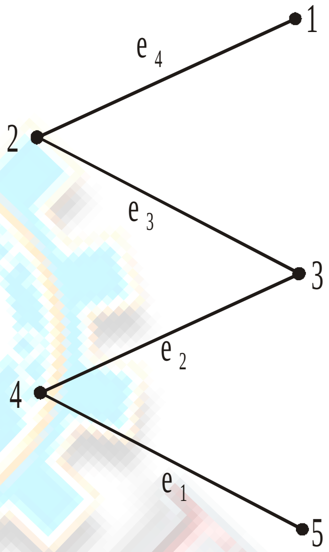
Hence ring sum can also be denoted as

$E_1 \Delta E_2 = (E_1 - E_2) \cup | (E_2 - E_1)$

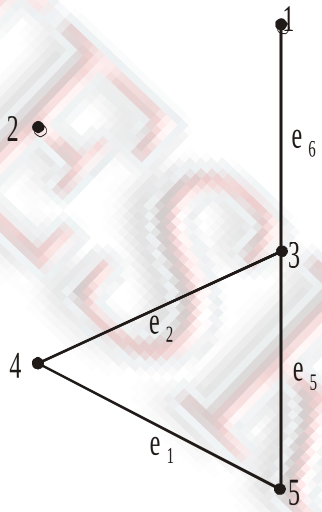
let us explain the above concept with the help of examples.

Example: let us consider two edges set E_1 and E_2. Let E_1 is containing four edges i.e. $E_1 = \{e_1, e_2, e_3, e_4\}$ and the other set E_2 also contains four edges i.e. $E_2 = \{e_1, e_2, e_5, e_6\}$. Now find the ring sum of E_1 and E_2.

The edge set E_1 is i.e.

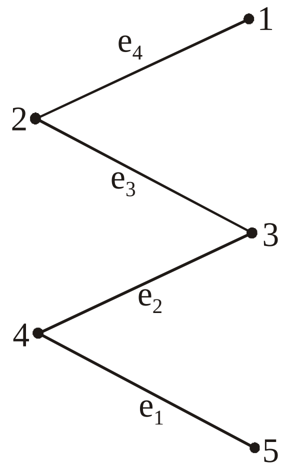


And the edge set E_2 i.e.

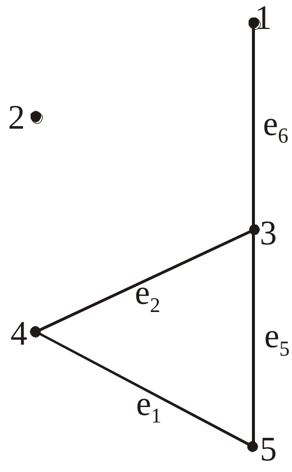


Now we can observe than in E_1 and E_2. There are two common edges e_1 and e_2.

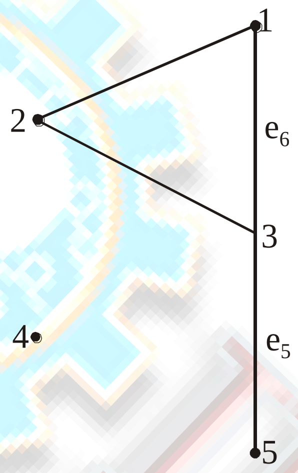
$E_1 \oplus E_2 =$



$\oplus$



=



REFERENCES

1. N. Alon, Restricted colorings of graphs. In K. Walker, editor, Surveys in combinatorics, number 187 in London Math. Soc. LNS, (1993), 1-33.
2. M. Atici, Computational complexity of geodetic set, Int. J. Comput. Math., 79 (2002), 587-591.
3. R.C. Brigham, P.Z. Chinn, R.D. Dutton, Vertex domination- critical graphs, Networks, 18 (1988), 173179.
4. J. C. Bermond, F. Comellas, D.F. Hsu, Distributed loop com-puter networks: a survey, J. Parallel Distrib. Comput., 24 (1995), 2-10.
5. R.C. Brigham, T.W. Haynes, M.A. Henning, D.F. Rall, Bicrit-ical domination, Discrete Mathematics, 305 (2005), 18-32.
6. F. Boesch, R. Tindell, Circulants and their connectivity, J. Graph Theory, 8 (1984), 487¡499.
7. W. E. Clark, L. A. Dunning, Tight upper bounds for the dom-ination numbers of graphs with given order and minimum degree, The Electronic Journal of Combinatorics, 4 (1997), #R26.
8. G. Chartrand, P. Zhang, The forcing convexity number of a graph, Czech. Math. J., 51 (2001), 847-858.
9. Ch. Eslahchi, M. Gebleh and H. Hajiabolhassan, Some concepts in list Coloring, Combinatorial Mathematics and Combinatorial Computing, 41 (2002), 151-160.
10. D. Fisher, K. Fraughnaugh, S. Seager, Domination of graphs with maximum degree three, Proceedings of the Eighth Quadrennial Internationa Conference on Graph Theory, Combinatorics, Algorithms and Applications, I (1998), 411-421.
11. A. Kemnitz, H. Kolberg, Coloring of integer distance graphs, Discrete Math., 191 (1998), 113-123.
12. Li-Da Tong, The (a,b)-forcing geodetic graphs, Discrete Math., 309 (2009), 1623-1628.
13. Li-Da Tong, The forcing hull and forcing geodetic numbers of graphs, Discrete Appl. Math., 309 (2009), 1623-1628.
14. A. P. Santhakumaran, P. Titus, J. John, The upper connected geodetic number and forcing connected geodetic number of a graph, Discrete Applied Mathematics, 7(157) (2009), 1571-1580.